

# Insight into the Theory of Relativity

ANALYSIS OF THE THEORY OF RELATIVITY FROM THE  
STANDPOINT OF TWO OBSERVERS THAT OBSERVE THE SAME  
PHYSICAL PHENOMENON .

Presentation of the online paper:

“Insights into the Theory of Relativity”

“part I”

“Basic Principles and Starting Points”

[www.relativityworkshop.com](http://www.relativityworkshop.com)

F.Sanchez-Martin

## INTRODUCTION

In this presentation I intend to develop new insights in the theory of relativity. I think I throw some new concepts in a new light.

For the moment, the main outcome I get is the deduction of Lorentz transformation regardless of some principles as the homogeneity and isotropy of space, rigidity of bodies, constancy of velocity of light and Maxwell equations.

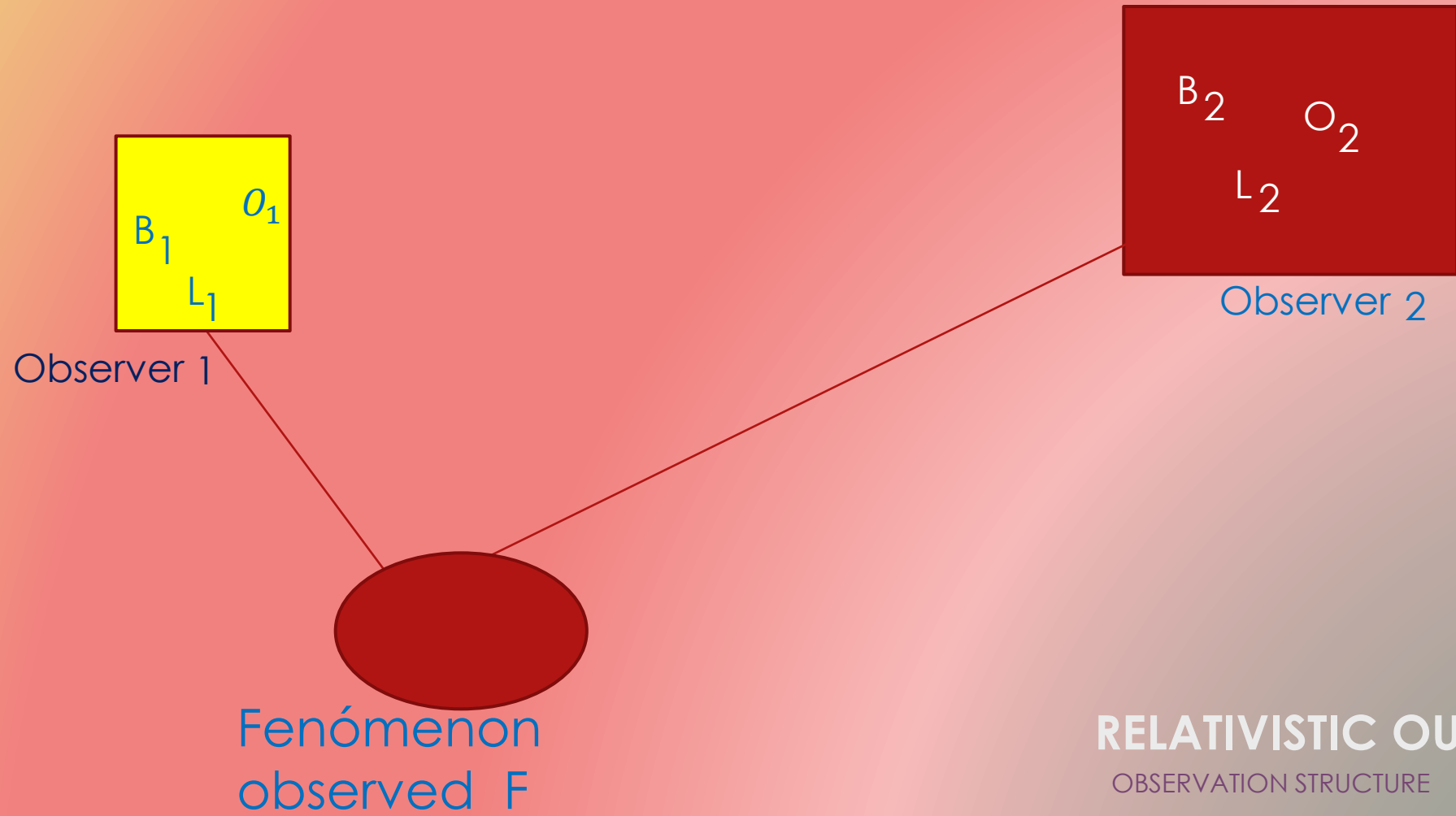
Partially I agree to the relativity principle.

We rediscover orthogonal transformation between two reference frames, namely, the Lorentz transformation, even if in a more general way.

On this basis it is derived Lorentz boost transformation for inertial reference frames .

From the beginning we are working on the basis of two observers that have their own space-time with the same features (namely, dimension, the same signature, etc..) everyone.

These space-times are real vectorial lorentzian space-times with signature  $(-1,1,1,1)$  endowed with a metric non degenerated, although, in the starting, the theory developed here is independent of the dimension and signature of the space-time of the observers.



**Fig. 1** Observation diagram . Observer 1,  $O_1$ , and Observer 2,  $O_2$ , observe and measure the physical phenomenon F.

The observer  $O_1$  (observer 1) and  $O_2$  (observer 2) observe the same physical phenomenon  $F$ .

Each observer has his own lorentzian vectorial space-time.

The space-time of observer 1 is  $L_1$


The space-time of observer 2 is  $L_2$

The observer 1 has a vectorial basis  $B_1$

The observer 2 has a vectorial basis  $B_2$

From a classical physical relativistic standpoint, vectorial basis are seen like three measuring roles and a clock.

Observer  $O_1$  and observer  $O_2$  observe the same physical phenomenon  $F$ .  $O_1$  gets the measures  $F_1$ , and  $O_2$  gets the measures  $F_2$ .



In this paper it is relevant to single out the known importance of electromagnetic field in the theory of relativity. Poincaré set up and gave a great relevance to the role that equations of Maxwell play in the theory of relativity. The electromagnetic field is so closely related to the theory of relativity that the theory of relativity is to be constructed out of the electromagnetic field structure.

That is because electromagnetic fields together gravitational fields are the only fields that make the macrocosm.

Actually electromagnetic fields penetrate deeply into the mechanics of the physical world.

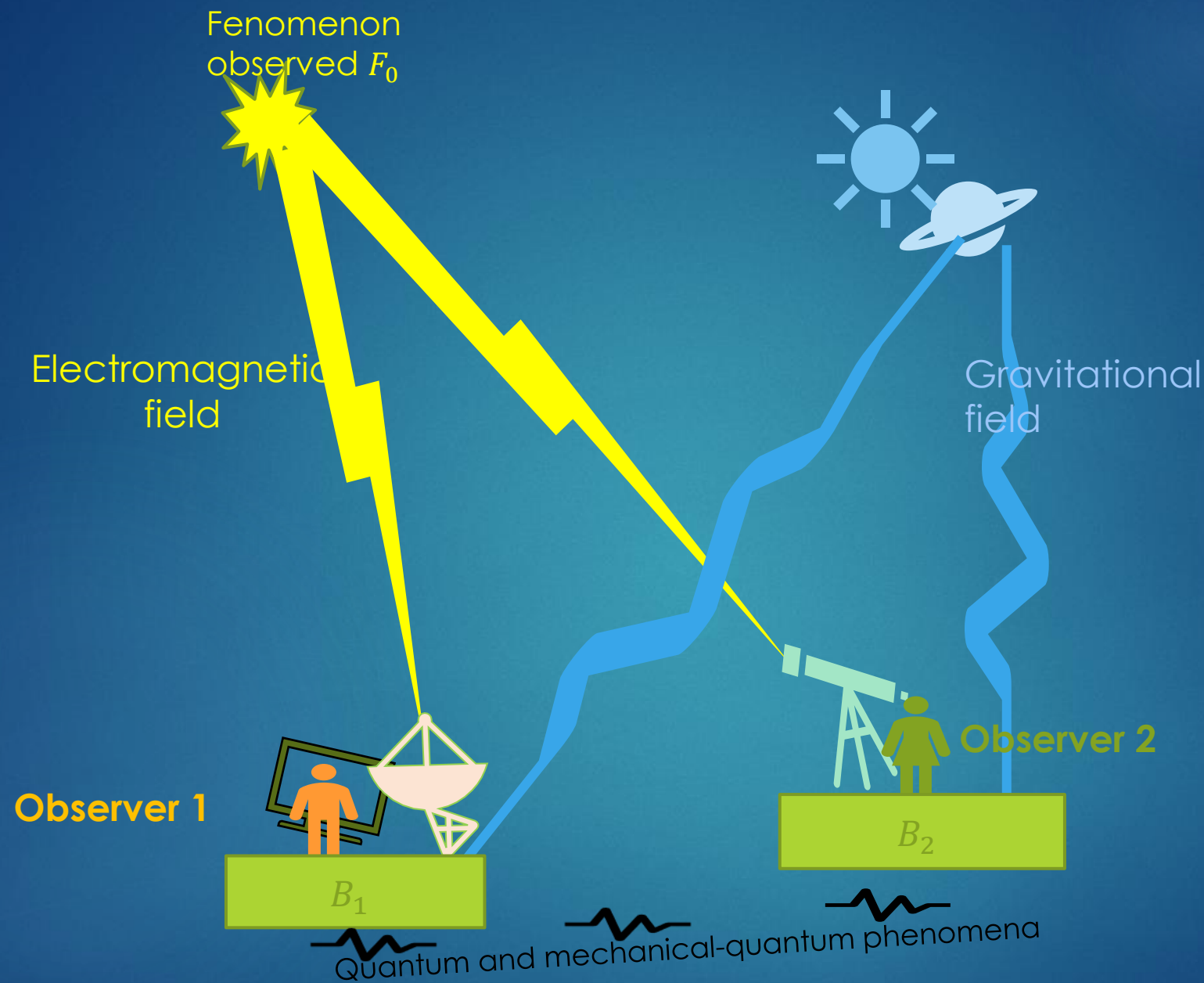


Fig.2- macroscopic fields

The dominant fields in the macrocosm are the electromagnetic field and the gravitational field.

The electromagnetic field is involved in a direct or indirect way in the physical phenomena of the macrocosm, (apart from gravitational). It determines the physical processes in the macrocosm.

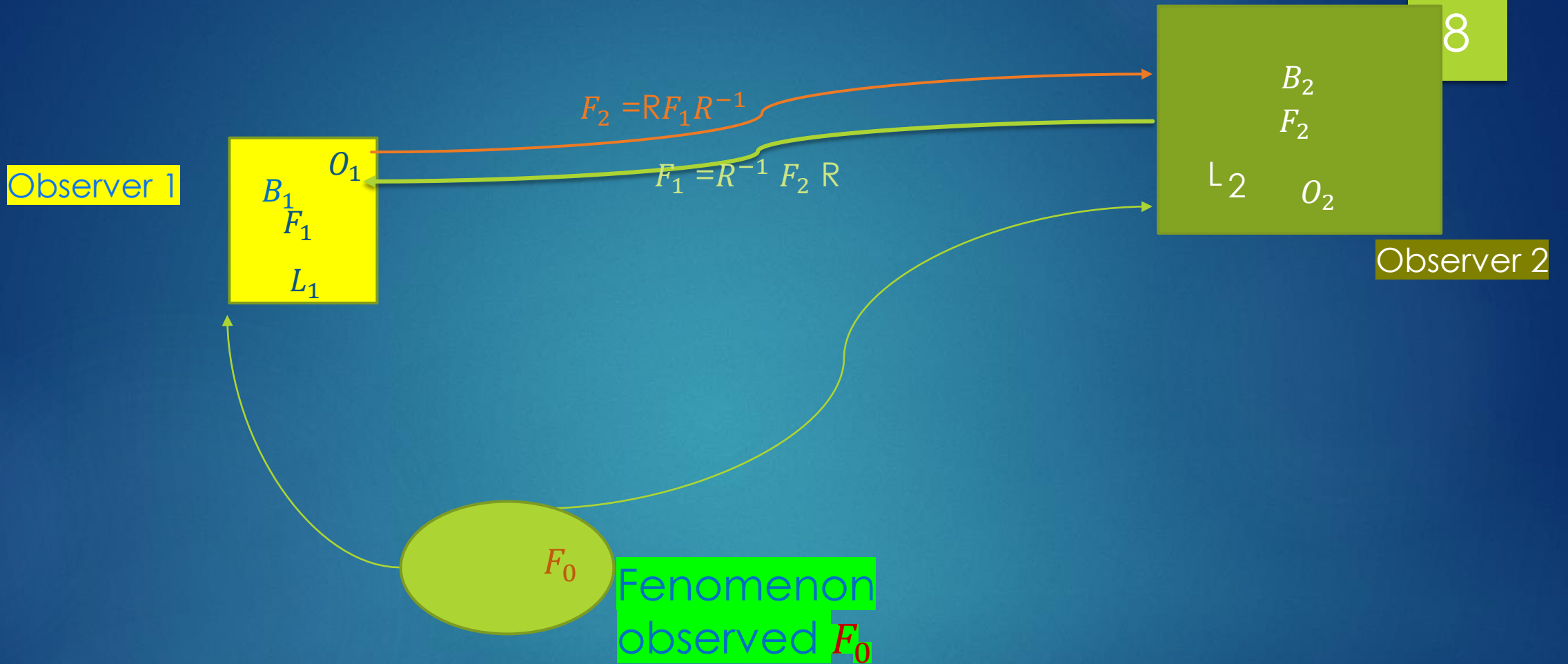


## Principle of coupling of observers.

This principle is:

Each observer (see fig. 1), has a base from which, both observers see the same phenomenon  $F_0$ , and get the same electromagnetics fields measures; (see fig. 2).

- ▶ Thereby, from whatever other two bases,  $B_1$  of observer  $O_1$ , and  $B_2$  of observer  $O_2$ , is derived a mathematical similarity relation  $\mathbf{R}$  between the matrices of components of  $F_1$  and  $F_2$  (see fig. 3); (this is proved in the online publication "Insights into the Theory of Relativity"; part I; pág. 5, subsection 2.1-Principle of coupling of observers; [www.relativityworkshop.com](http://www.relativityworkshop.com)).
- ▶ Therefore, we have an homomorphism  $\mathbf{R}$  between the matrices of measures of  $F_1$  and  $F_2$  :
- ▶  $F_2 = \mathbf{R}F_1\mathbf{R}^{-1}$  .



**Fig. 3** Diagram of observation . Observer 1,  $O_1$ , and observer 2,  $O_2$ , coupled. They observe and measure the same physical phenom  $F_0$ .

Coupled for us, mean:

Everyone observer has a base from which both observers , observe and measure the same physical phenomenon  $F$  , **getting the same measures**. We call it coupling of observers.



# Electromagnetic field structure.

- ▶ Then, we begin on the next basis:
- ▶ The electromagnetic field is a second order antisymmetric tensor, in a Lorentzian vector space, in accordance with the structure currently involved in relativistic physics (see Note 2).

This antisymmetric structure must be preserved in any transformation to hold the physical meaning of the electromagnetic field.

Note 2-

$F_i$ ,  $i = 1, 2$  are second order skew-symmetric tensors in a vectorial lorentzian spacetime. In this context, Maxwell equations in the empty space, in classical relativity, are:

$$d_{ext}F_i=0 ; d_{ext}F_i^*=0 ; i = 1, 2 ;$$

where  $*$  is the Hodge operator and  $d_{ext}$  is the differential exterior.

# Orthogonality between the space-times of observers.

10

## Definition:

The homomorphism  $R$  between the space-time  $L_1$  of  $O_1$ , with metric  $G_1$ , and the space-time  $L_2$  of  $O_2$ , with metric  $G_2$  is orthogonal if the scalar product is preserved and viceversa. That is:

$$G_1(X_{o_1}, Y_{o_1}) = G_2(X_{o_2}, Y_{o_2}); \quad \forall X_{o_1}, Y_{o_1} \in L_1 ; \quad \forall X_{o_2}, Y_{o_2} \in L_2, \text{ being:}$$
$$X_{o_2} = R(X_{o_1}); \quad Y_{o_2} = R(Y_{o_1})$$

- Notice the concept of homomorphism between two different vectorial space-times differs of the concept of homomorphism into a vectorial space (also called endomorphism).
- In the first case the linear mapping is established between vector components only. It is not a mapping between base vectors. It is a **coordinates transformation**.

**R** is transformation  
between space-  
times  $L_1$  and  $L_2$

If tensors  $F_1$  and  $F_2$   
are skewadjoint then  
it is proved that **R** is  
to be orthogonal.

This is taking into account that  
fields  $F_1$  and  $F_2$  are the observed  
electromagnetic fields and these  
fields are measured by observers  
 $O_1$  and  $O_2$ .

In short, the homomorphism **R** that  
**preserves** the features  
(skewadjoint) of electromagnetic  
tensor fields **is to be orthogonal**.

- ▶ See the prove of this assumption in
- ▶ "Insights into the Theory of Relativity" ,; Part I;  
P. 7 and 8, subsection 3.1-Principle of  
coupling of observers;  
[www.relativityworkshop.com](http://www.relativityworkshop.com)).
- ▶ It is previously proved that an orthogonal  
transformation **R** keeps the tensor operation  
of adjoint endomorphism.
- ▶ Note 3- If the tensor is symmetric, the  
symmetrical characteristic of the tensor is  
also preserved in an orthogonal  
transformation.

## Reduction of observer space-time to an affine space

In the beginning, following the perspective of classical relativity, we establish that observers and the observed phenomena are immersed into a space-time of points  $\mathbf{O}$ , associated to a lorentzian vectorial space-time  $\mathbf{L}$ .

That is  $O_1$ ,  $O_2$  and  $O_0$  are points of an affine space.

Within this context,  $O_0$  is the point linked to the observed phenomenon  $F_0$

- ▶ Their reference frames are represented into the associated vectorial space  $\mathbf{L}$ .
- ▶ **Now, in this way and into this context,  $L_1$  and  $L_2$  can be treated as subspaces of  $\mathbf{L}$**
- ▶ **Now,  $R$  acts as an endomorphism into  $\mathbf{L}$ .**
- ▶ **The bases  $B_1$  and  $B_2$  now can work into  $\mathbf{L}$ , as well.**
- ▶ See fig. 4



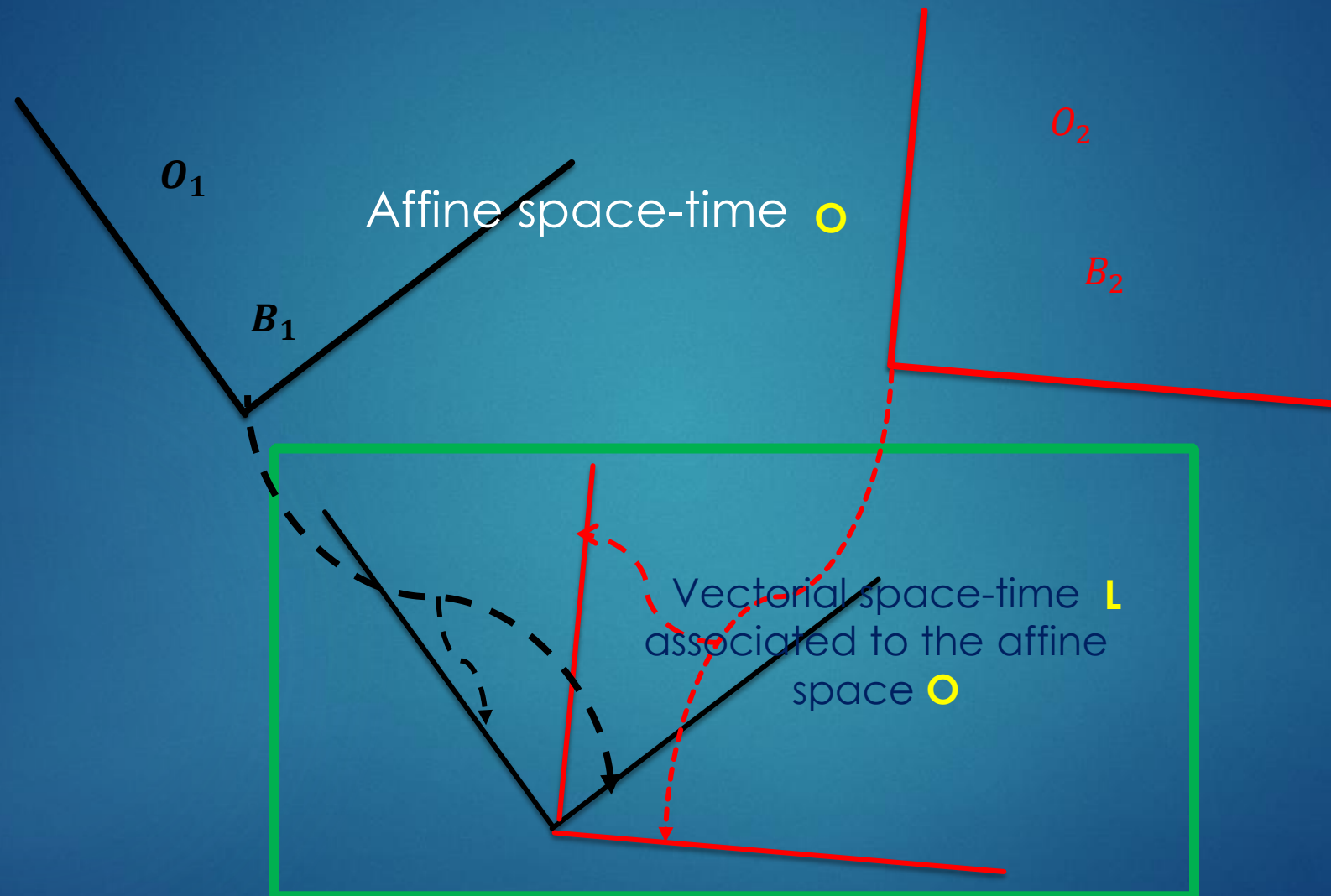


Fig. 4 . Affine space-time and associated vectorial lorentzian space



Restricting to the field of the classical theory of special relativity, that is, to inertial reference frame, we have:  $G_1 = G_2 = G$  ;

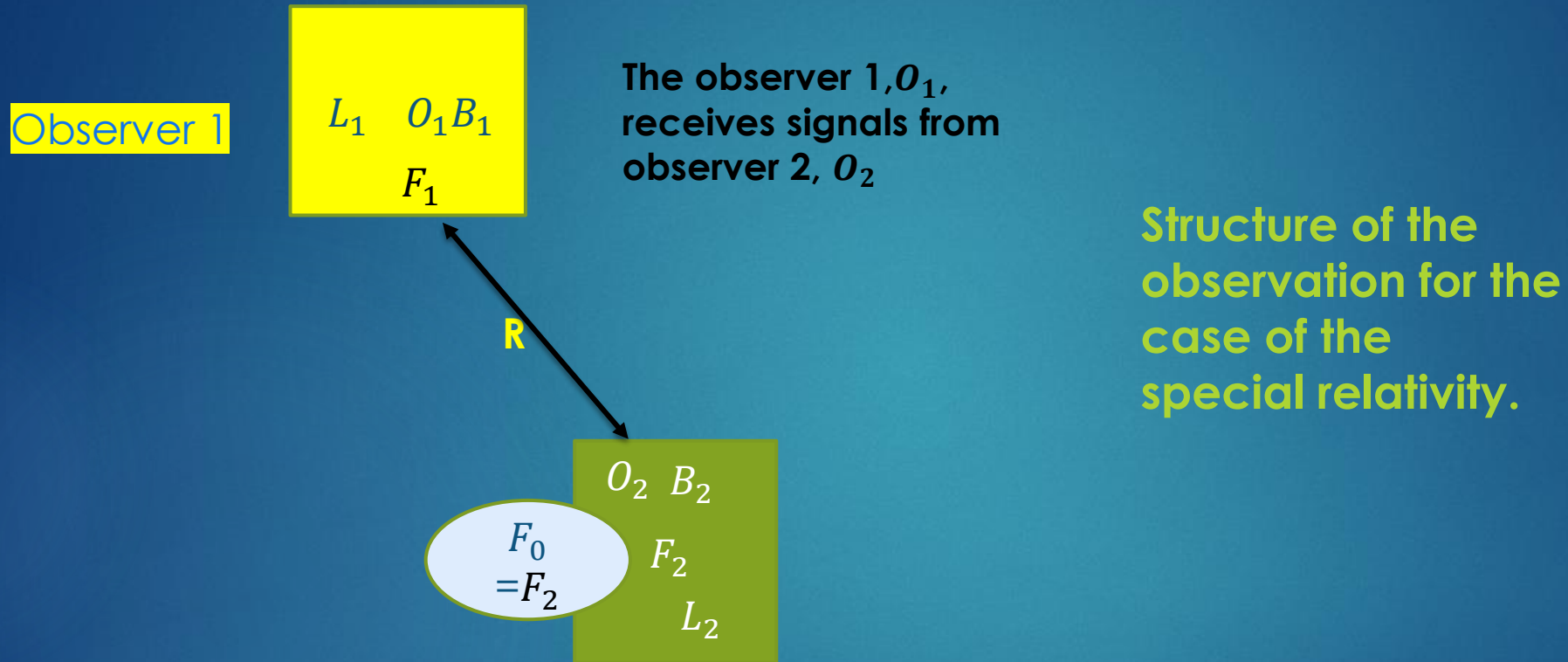
where universe Interval is

$$s^2 = -x_0^2 + x_1^2 + x_2^2 + x_3^2$$
$$x_0 = c \cdot t$$

Besides  $E_2$  coincides with  $E_0$ . Or rather the observer  $o_2$  coincides with the observed phenomenon.

See Fig, 5.

$$s^2 = -c^2 t^2 + x_1^2 + x_2^2 + x_3^2$$



Observed phenomenon  $\equiv$  Observer 2

**Fig. 5** Observation diagram.

The observer 2,  $O_2$ , coincides with the observed physical phenomenon  $F_0$ .

# Steadiness of the speed of light.

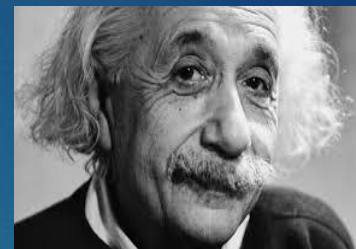
We shall use similar reasoning to those of special relativity.

The metric is:

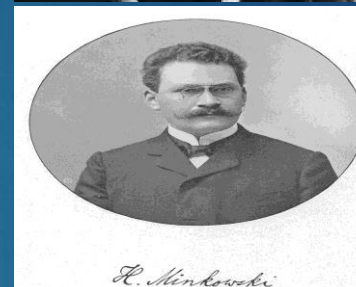
$$s^2 = -\mathbf{c}^2 t^2 + x_1^2 + x_2^2 + x_3^2$$

The orthogonal transformation  $\mathbf{R}$  leaves invariant the skew-adjoint feature of electromagnetic field.

It leaves invariant the coefficients of the metric, as well. Among them  $\mathbf{c}$ , the velocity of the light, remains invariant.



Einstein



Minkowski



Lorentz



Poincaré

In this way we attain the starting point of special relativity: **The constancy of the speed of light for inertial reference frames.**

17

- ▶ So, into a context of space-time and from the tensorial and skewadjoint features of electromagnetic fields, we infer the next two points :
- ▶ a)—Between two inertial reference frames there is an orthogonal transformation or rather, Lorentz transformation.
- ▶ b)—The speed of the light remains constant for inertial reference frames.



# The electromagnetic field and the special theory of relativity

18

## Electromagnetic field and the Lorentz transformation

**The basis of the special theory of relativity is the electromagnetic field, according with the Poincaré-Einstein starting points.**

- ▶ Thereby, the theory of special relativity is only valid in the domain in which are involved the electromagnetic fields.
- ▶ The Lorentz transformation leaves invariant the skewadjoint structure of electromagnetic field.

## Other non electromagnetic fields

- ▶ In the domains in which there are not electromagnetic fields, the theory of special relativity is not fulfilled.
- ▶ When the conclusions of special relativity are not fulfilled, we must rule out, for example **the equivalence between matter and energy**, in the domain of the aforementioned non-electromagnetic fields.

However, as we saw earlier, (see pag 11, note 3) , there are other fields, as symmetric tensor fields, that are invariants under an orthogonal transformation, or rather, under a Lorentz transformation. This would be applicable to stress energy tensor.



# The electromagnetic field and the theory of relativity

## Electromagnetic field and Lorentz transformation

The theory of special relativity is basically a theory for the electromagnetic field..

- ▶ Some texts develop the theory of relativity from several postulates without taking into account the electromagnetic field; see for example "The Classical Theory of Fields"; L. Landau and E Lifshitz; Chapter I.
- ▶ However, at the end, **the constancy of the speed of light appears**. That is, the intervention of the electromagnetic field. And it also appears as a maximum limit.

## Other fields not electromagnetics.

- ▶ If other fields appear in the macrocosm, it would be necessary to look for new transformations that conserved their characteristics (in a suitable space-time frame).
- ▶ The physical interpretation of these transformations would lead us to find out another theory, counterpart to the relativity. This would be applicable only in areas of the macroscopic universe where such fields exists.

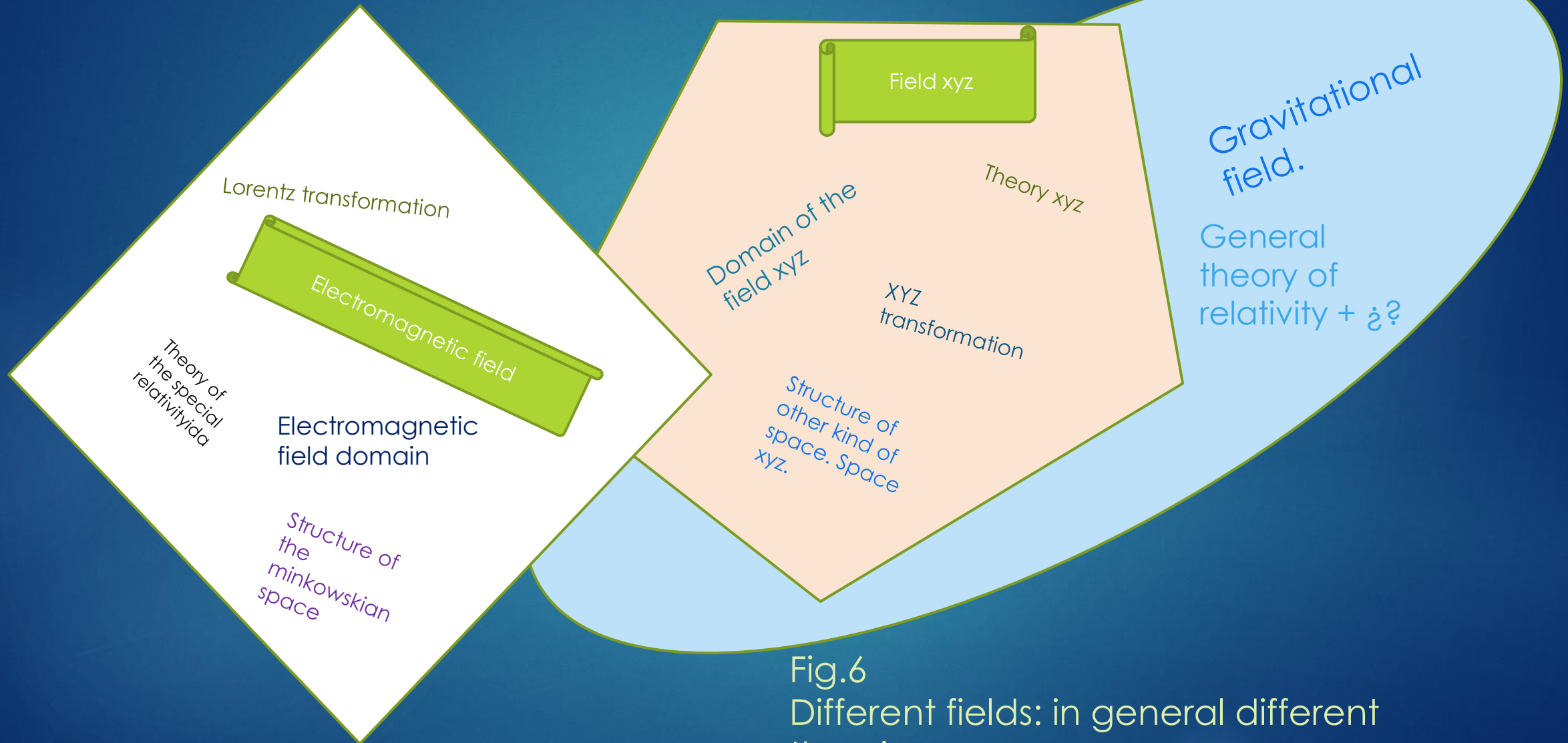


Fig.6  
Different fields: in general different theories.

The Minkowskian space-time structure is suitable for the description of the electromagnetic field. In another structure, Maxwell's equations could not be represented with all their characteristics and invariances.

In the intermediate between macrocosm and microcosm, that is, for example at molecular scale, the theory of relativity could be applied with caution.

It is possible that other non-electromagnetic fields would have to be represented in different space-time structures, counterpart to the space-time structure that represents the electromagnetic field in relativity.

The microcosm is another different world. In it, quantum physics is applied with totally different concepts and structures.

In neutral media (formed by particles composed of a charge + and another -, (that is, formed by particles with zero resulting charge), such as molecular atomic media, electromagnetic fields are made up and the action of these fields is vanished.

However, the components of these fields exist. Therefore they are transformed by the Lorentz transformation.

Therefore in the neutral media (charge = 0) the Lorentz transformation is applicable.